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Mapping Speed

Summary We estimate the sensitivity of a bolometer array when measuring the flux of an isolated point source whose location is known in advance.

Introduction We might estimate *SCUBA2*'s sensitivity by measuring the rms voltage noise in a one second average and the change in response looking at a thermal loads which differ by ΔT. From this we would get a Noise Equivalent Temperature expressed in $K\sqrt{s}$. Alternately, we could measure the voltage response when a point source of known strength S is directly on axis and describe our noise as a Noise Equivalent Flux Density, ***NEFD***, expressed in $Jy\sqrt{s}$. The variance of a measurement using a single bolometer whose duration is t, taken without moving the telescope, is

$$\sigma_t = \frac{NEFD}{\sqrt{t}} \tag{1}$$

as long as t is longer than our detector response time. At shorter times the signal is correlated and Eq 1 overestimates the variance.

Assuming we know the $NEFD$ for the *SCUBA2* bolometers, let's work out the mapping speed for scanning an area of sky simultaneously with N_B bolometers for a time t_{Obs}. Just to be quite explicit, we will assume a pixel size, find the variance per pixel and then fit the pixel values to infer the amplitude of an isolated point source. The pixel size drops out in the end as long as it is less than the optical point spread function and at the same time large enough that it is visited often enough that the variance ca be inferred from Eq 1. If the solid angle of the map is denoted by Ω_M, and that of an individual pixel is $\theta_p^2 = \Omega_p$. the average observation time per pixel is

$$t_p = \frac{t_{Obs} N_B \Omega_p}{\Omega_M} \tag{2}$$

and so the variance per pixel is

$$\sigma_p = NEFD\sqrt{\frac{\Omega_M}{\Omega_p N_B t_{Obs}}}. \tag{3}$$

Notice that the beam shape does not enter explicitly into the expression for the σ_p, the pixel variance. It is, however, hidden in the value of the $NEFD$ because the on-axis response to a point source is low if the beam is wide. For the careful, please also notice we are assuming a complicated enough scan pattern that the noise in adjacent pixels is not correlated and a long enough observation that we can characterize the value of each pixel in a map as having a variance.

Mapping Speed What is the noise on our best estimate of the flux of a point source at p if we have a map in which each pixel has variance σ_p? We estimate source brightness, S_o, by minimizing χ^2 for fitting the telescope point spread function to the map.

$$\chi^2 = \sum_j (S_j - S_o e^{-(j-p)^2\theta_p^2/2\theta_B^2})^2 \tag{4}$$

where $(j-p)\theta_p$ denotes the angle between pixels j and p and θ_B describes the gaussian optical beam width, $PSF \propto e^{-\theta^2/2\theta_B^2}$. The pixel index j runs over a full 2D map larger than the point spread function.

For the moment, write θ_B/θ_p as η. The most likely value of S_o given the data is obtained from solving

$$\frac{\partial\chi^2}{\partial S_o} = 0 = -2\sum S_j e^{-(j-p)^2/2\eta^2} + 2S_o \sum [e^{-(j-p)^2/2\eta^2}]^2 \tag{5}$$

so

$$S_o = \frac{\sum S_j e^{-(j-p)^2/2\eta^2}}{\sum [e^{-(j-p)^2/2\eta^2}]^2} \tag{6}$$

The variance on S_o can be expressed in terms of the variance per pixel and the beam size as

$$\sigma_S = \sigma_p \frac{1.}{\sqrt{\pi}\eta} = NEFD\sqrt{\frac{\Omega_M}{\Omega_p N_B\, t_{Obs}}}\ \frac{1.}{\sqrt{\pi}\eta} \tag{7}$$

and here we can explicitly cancel out the pixel size:

$$\sigma_S = NEFD\sqrt{\frac{\Omega_M}{N_B\, t_{Obs}}}\ \frac{1.}{\sqrt{\pi}\theta_B}. \tag{8}$$

That is our mapping speed. It gives the variance of the inferred flux of a point source if you know the observation time, the Noise Equivalent Flux Density, the beam size, and the number of bolometers.

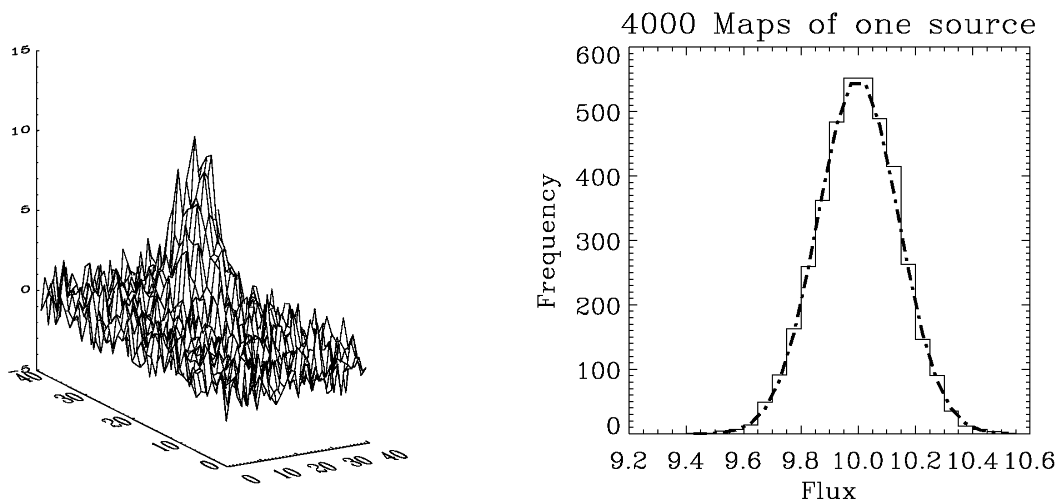

Figure 1: Simulations were made of our mapping speed for isolated point sources by producing 4000 individual 40pixel×40 pixel maps with unit variance and a 10 mJy point source at [20,21]. The point spread function (PSF) corresponds to $\eta = 4$. The image to the left is a single realization. THe image to the right is a histogram of the flux inferred from the individual maps using Eq. 6. A gaussian corresponding to Eq 8 is overplotted with a dot-dashed pattern.